sketch of a "four-way partition" of the flow or equivalently a saddle-point streamline configuration was included in Ref. 5. Trifurcation of the flow is certainly unrealistic, and I know from my discussions with Moore and his co-workers that their sketches simply did not show any details in the wake region. They did not propose a trifurcation configuration.†

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†Note added in proof: Communicating privately with Dr. O'Brien I resolved my misunderstanding with regard to "trifurcation of the flow." By this term we understand a point where three streamlines meet which is not possible unless the two of the streamlines coincide with a solid boundary. Dr. O'Brien on the other hand implies a saddle point streamline configuration where two of the streamlines form a closed loop thus partitioning the space into three aeral domains.

Reply by Author to D. P. Telionis

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MY article[1] presents a particular internal flow problem, but I fully intended that it apply as an example for unsteady separation phenomena from two-dimensional solid bodies in a variety of flow situations. Discussion of unsteady flow procssses,[2] as in many flow processes that are not completely understood, suffers from semantic problems. As used in my article, detachment and reattachment apply to all (instantaneous) streamlines that intersect the solid boundary, whatever the characteristic Reynolds number, internal or external flows. A streamline having both a detachment point and a reattachment point is, per force, a closed recirculation region. In aerodynamic parlance, this is a "separation bubble". It may be thick or thin relative to the upstream boundary layer. If it is thick, it appears the boundary layer is breaking away from the body. Yet true boundary-layer "breakaway"[3] is generally reserved for open separated regions that merge with the wake without reattachment to the body; this is called "separation" in Ref. 2 and sometimes "blow-up" in Ref. 4. Common imprecise use of the term "boundary-layer separation" lumps together thin and thick separation bubbles along with true breakaway as if there were no distinction. In each case, the local flow patterns near detachment and/or reattachment points can only be accurately revealed by regular full Navier-Stokes solutions. The common feature is *coincidence of velocity and vorticity zeroes*. This amounts to the usual "vanishing wall-shear" for steady flow, but the criterion applies as well to unsteady shear flow, though not yet properly accepted. We could also speak of mean detachment points and reattachment ones for pulsatile or turbulent flows.

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Steady boundary-layer theory has been applied successfully to high Reynolds number external flow problems. Yet the distinction between the apparent singularity of the boundary-layer solution and regular behavior at detachment for the full momentum equation[5] is well-known. (The latter is more accurate, of course.) Likewise the use of unsteady boundary-layer equations,[2,4,6] though used for practical engineering estimates for high Reynolds numbers, cannot be accurate for local flow details near detachment. Such analyses are always incomplete in the sense of Riley (Ref. 7 p. 283). The claim in Ref. 2 that a point of detachment (i.e. "my separation") has no major engineering significance is arguable, because it must always precede the bulk flow reversal region (thick or thin, open or closed, transient or permanent. Such flow regions can seriously affect heat or mass transfer.

Finally, if the body surface is moving forward in the frame of reference, the stagnation-separation point (in the Eulerian description) must occur off the body in the freestream. Zero wall-shear (on the body surface) predicts nothing about the streamline intersection. The "saddle-point streamline configuration" (Ref. 4, Fig. 1a) is my four-way partition (Points C & D, Fig. 2, Ref. 1). However, generally the orthogonal intersection need not be parallel-normal with respect to the solid wall. On the other hand, the unsteady shedding of vortices as revealed by full momentum equation calculations[8,9] does not involve such intersections but an osculating streamline ('trifurcation') where the vorticity is not zero. This is clearly a different thing.

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Comment on "Theoretical Study of Lift-Generated Vortex Wakes Designed to Avoid Rollup"

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RECENTLY Rossow[1] introduced two hypothetical vortex wakes and explored, through the use of the discrete-

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vortex model, whether the rollup of lift-generated vortex sheets can be suppressed. He has predicted a substantial reduction in rolling moment for wings that encounter the wakes that trail from span loadings with a sawtooth character. The basic premise of the proposal is that wakes comprising many counter-rotating vortices are more apt to produce erratic motions which diffuse the vorticity than a single pair of stronger vortices.

An essential feature of the discrete-vortex model is that the larger the number of the point vortices and the vortex sheets the larger the error growth during the computation. Rossow and the references cited by him discussed the shortcomings of the method. Similar computational difficulties in the use of the discrete-vortex model have been encountered by this writer[2,3] and resolved through the use of various techniques which help to suppress the vortex excursions and sheet kinking. Such techniques, as pointed out by Rossow, contain arbitrary parameters that are not related to the conservation equations for the fluid they are to represent. The purpose of this Note is to point out that *the primary source of difficulty is in not rediscreticising the sheet at each time interval of the calculation* and that it is not possible to calculate the motion of a vortex sheet from the motion of the series of discrete vortices used to represent that sheet for more than infinitesimal time intervals.

Fink and Soh,[4] in a report published about six months prior to the receipt of the revised version of Rossow's paper,[1] have shown that the complex conjugate velocity $\bar{q}(z_j)$ at a point z_j, on the vortex sheet, due to series of n discrete vortices is given by

$$\bar{q}(z_j) = \frac{1}{2i\pi} \sum_{k \neq j}^{n} \frac{\Gamma_k}{z_j - z_k} - \frac{1}{2i\pi} \frac{\Gamma_{je}{}^{-i\theta}j}{|s_{j+1/2} - s_{j-1/2}|} Ln \left| \frac{z_j - z_{j+1/2}}{z_j - z_{j-1/2}} \right|$$

The point z_j *lies with the segment* $(s_{j+1/2}, s_{j-1/2})$, without necessarily bisecting it at all times, and s measures the distance along the sheet. The remarkable consequences of this expression are that: a) if the equivalent vortex is not placed at the mid-point of its segment through rediscretization of the sheet at each time interval, then the logarithmic term does not vanish and the computational error increases depending on the problem, the number of vortices and time interval used, and the total time of computation; b) the vortices which initially bisect the segment which they are to represent do not continue to do so at the succeeding time intervals; c) the use of finite vortex cores, accumulation of vortices at the center of the spiral, or other techniques only delay or minimize the accumulation of the errors resulting from the logarithmic term in an amount related to the distance between z_j and the center of the segment; and that d) the growth of the computational error may be significantly reduced by placing each discrete vortex at the mid-point of its segment, i.e. by placing the vortex at $z_j = 0.5(z_{j-1/2} + z_{j+1/2})$ at each time interval. Only through such a procedure that one can make the logarithmic term vanish.

The calculations are then carried out at each time step by representing the vorticity density by an entirely new set of equi-distant vortices whose strengths are adjusted to give a good representation of that density. Evidently, this procedure does not resolve all of the computational errors particularly in regions where the curvature of the sheet is small, e.g., the region close to the center of the vortex spiral. Furthermore, the curve-fitting errors incurred in the process of interpolation at every time step may accumulate as time increases. Nevertheless, Fink and Soh[4] have shown through several examples that the rediscretization method overcomes many of the difficulties encountered by the previous users of the discrete-vortex model.

This writer has applied the rediscretization or the resegmentation method to the solution of the flow past an inclined plate (already worked out by him[3] through the use of the discrete vortex model without rediscretization) and found that: a) rediscretization could be used only for relatively small times ($Ut/s < 1$); b) the forces acting on the plate predicted without without rediscretization are practically identical; c) the decreasing separation between the turns of the spiralling sheet causes interaction and orbiting between the adjacent vortices and requires segment lengths smaller than the separation distance. This, in turn, considerably increases the computation time even for $Ut/s < 1$.

Evidently, rediscretization can be effective under certain circumstances. It is not, however, ready to replace the current techniques of application of the discrete-vortex model. Considering all other alternatives for the calculation of separated flows for which the wake is not replaced by a dead body of fluid, the discrete vortex approximation can produce numerically stable and viable solutions through the use of judiciously selected parameters such as the number of vortices, time interval during which the vortices are convected, etc. Rossow, like many others who have used this method, seems to have struck such a balance in his exploratory calculations of a rather complex problem.

The idea of tailored loading is an interesting one and may lead to the reduction of wake-vortex hazard. As such, it deserves further analysis with evenly as well as unevenly spaced sawtooth configurations. However, it remains to be seen whether the tailored loading will prove to be more practical and superior to other methods proposed such as the use of a 2500 HP engine at each wing tip of a DC-10 which is to prevent, through its counter rotating flow, the formation of the concentrated vortices without affecting the lift-to-drag ratio. Model studies based on this concept are currently being carried out by the Garrett Corp. at Pheonix, Arizona.[5]

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Reply by Author to T. Sarpkaya

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THE Technical comment by T. Sarpkaya calls attention to a new method derived by Fink and Soh[1] for predicting the rollup of vortex sheets. This method differs from others in that it contains a logarithmic term in the ex-

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